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THE PRICING AND RECEPTION OF BONDS EXTENDIBLE AT THE OPTION OF THE GOVERNMENT OF CANADA

A study prepared for the Canada Mortgage and Housing Corporation

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EXECUTIVE SUMMARY

1. This paper analyzes the likely market reception of a 5-year bond extendible (at the initial coupon rate) for an additional 5 years at the option of the Government of Canada.

2. This proposed extendible bond is equivalent to the holder owning either (1) a straight 5-year bond less a put option or (2) a straight 10-year bond less a call option.

3. From the viewpoint of implementing Pesando-Turnbull, the objective is to estimate the yield premium required on the proposed bonds relative to straight 5-year bonds. The put option held by the Government in the proposed bond is equivalent to that held by the holder of a 5-year bond extendable for 5 years at the holder's option, relative to a straight 10-year bond.

   (1) If symmetry prevails, the yield premium required on the proposed bond will equal - at each point in time - the excess of the yield on a straight 10-year bond over that on a present 5-year extendible bond when the latter is trading at par.

   (2) Based on 12 pairings of existing extendible and straight 5-year bonds, this yield premium ranged from 11 to 90 basis points, and averaged 41 basis points.

   (3) These yield premiums reflect the prevailing term structures as well as the (unobservable) degrees of uncertainty regarding the path of future interest rates.

4. The lower bound on the yield premium on the proposed bond relative to a straight 5-year bond is the larger of (i) the excess of the yield on a straight 10-year bond over the yield on a straight 5-year bond and (ii) zero.

   (1) The following "rule of thumb" is suggested by the historical data: ADD 31 BASIS POINTS TO THE LOWER BOUND TO ESTIMATE THE REQUIRED YIELD PREMIUM.

   (2) As of 1 May 1983, this "rule of thumb" would suggest a yield premium of about 105 basis points.

   (3) This "rule of thumb" provides an estimate that is consistent with those supplied by respondents to our questionnaire, which were 75-100 and 125 basis points. Of these two estimates, the 75-100 appears more likely to represent a consensus view.

   (4) The fact that this estimate is high relative to the mean historical value of the equivalent put option probably reflects, in the main, the relatively sharp rise in the term structure that prevails at present.

5. Members of the investment community made the following additional points:

   (1) The proposed bond should be advertised as a 10-year bond callable after 5 years.

   (2) If the proposed bond were extendible at a coupon rate that exceeds the initial coupon rate, the required yield premium would be less. Respondents were not willing to estimate by how much.

(3) The reaction of potential buyers was characterized as guardedly positive, but not enthusiastic.

(4) If possible, the bonds should allow the option to be stripped and traded separately.

(5) Respondents priced the extendible bond by adding an estimate of the value of the call option to the straight 10-year yield. Their estimates of the value of this call option were 25-50 and 50 basis points. In our analysis of existing extendible relative to straight 5-year bonds, we estimate a mean historical value of 41 basis points for the equivalent call option.

6. Given the positive tone in the bond market, it would appear to be a favourable time to float a trial issue of the proposed bonds.

1. Introduction

The Pesando-Turnbull proposal for mortgage rate insurance centers on the issuance by the C.M.H.C. (or the Government of Canada) of bonds which are extendible at the issuer's option. For simplicity, let us assume that the C.M.H.C. is interested in insuring that a 5-year mortgage upon renewal bears an interest rate no more than 2% above the current 5-year rate. The C.M.H.C. would then be interested in issuing a 5-year bond, extendible at its option for an additional 5 years at an interest rate that is 2% above its initial coupon rate. There are two related questions which need to be addressed. First, what additional yield would market participants require on this extendible versus a straight 5-year bond? This yield spread is the market measure of the annual cost of the mortgage rate insurance. Secondly, how would market participants respond to the introduction of this type of instrument by the Government of Canada? The purpose of this report is to address these two questions.

There are two possible approaches to the problem at hand. The first is to analyze the manner in which the market has priced existing extendible and retractable bonds, in which the option to extend or to retract is held by the bond holder, relative to straight bonds. If options are priced symmetrically, so that the price is independent of whether the option is held by the issuer or bond holder, this approach should provide direct evidence on the excess yield required on the proposed extendible relative to straight 5-year bonds. The second approach is to survey the opinions of major underwriters on how the proposed bonds would be priced by the market. Since the underwriters

will contact ultimate purchasers ("accounts") in the course of their enquiries, additional information on the marketability of the bonds should also be generated. The question of symmetry would be addressed, either explicitly or implicitly. Both approaches are adopted in this study.

The first part of the study thus examines the historical premiums associated with extendible, retractable, and callable bonds. The study examines the pricing of these bonds in comparison to conventional bonds of the appropriate maturity. This information is used to infer the magnitude of the premium required on a bond which is extendible at the option of the issuer. The second part of the study reports the results of a questionnaire sent to major bond underwriters in Canada. The questionnaire is designed to obtain information about the pricing of the proposed bonds and the factors which may affect the market's reception of these bonds. The last part of the study provides a summary.

2. History of Extendible, Retractable, and Callable Bonds

As discussed at length in Appendix A, extendible, retractable and callable bonds are equivalent to straight bonds plus or minus a put or call option. These analytical results are used in our analysis of the pricing of extendible and retractable bonds relative to straight bonds of the appropriate maturity.

If the Government of Canada were to issue a 5-year bond extendible at its option for an additional 5 years, the holder of this bond would have the equivalent of a straight 5-year bond less a put option. The value of this put option could be most easily determined by simply comparing the price of the extendible bond with the price of a straight 5-year bond

with the same coupon. The price of the extendible bond would be less, and the difference would represent the market price of the put option that is held by the issuer.

For the purpose of implementing the Pesando-Turnbull proposal, our interest is not in the price of the put option per se, but rather in its annual or amortized cost. If a 5-year bond extendible for 5 years at the option of the issuer is priced at par with a coupon rate of 11%, while the straight 5-year bond is priced at par with a coupon rate of 10½%, then the annual or amortized cost of the put option is ½% each year. This ½% yield spread serves as the market measure of the annual insurance premium in the Pesando-Turnbull proposal. Since we are interested in establishing the likely size of this insurance premium, we are concerned with the relevant yield spreads - not the price of the put option per se - throughout our analysis.

(i) Extendible Bonds Issued by the Government of Canada

Between March 1980 and September 1982, the Government of Canada issued 13 bonds extendible at the holder's option (Table 1). The typical bond had an initial maturity of 5 years, and could be extended at the holder's option for an additional 5 years at the initial coupon rate. To the extent that the C.M.H.C. is interested in providing rate insurance for 5-year mortgages, the C.M.H.C. is interested in marketing 5-year bonds which are extendable at the issuer's option for an additional 5 years.

As explained in Appendix A, a 5-year bond extendible at the holder's option for an additional 5 years is equivalent to the holder's owning a straight 10-year bond plus a put option. A 5-year bond which is extendible at the issuer's option for an additional 5 years is equivalent to the

holder's owning a straight 5-year bond less the identical put option. Assume that the price of this option is symmetric and thus does not depend upon whether the issuer or the holder owns it. Then the excess of the yield on the straight 10-year bonds over the yield (when at par) on the 5-year bonds extendible at the holder's option provides the market measure of the annual or amortized cost of this put option. This measure, in turn, is the annual insurance premium in the Pesando-Turnbull proposal for mortgage rate insurance.

Worksheets for each of the extendible bonds identified in Table 1 are reproduced in Appendix B.* On each worksheet, the yield on the extendible bond at or near its issue date (when it is at or near par) is compared to the yield on both a straight 5-year and a straight 10-year bond. The yield comparison with the 10-year straight bond is of particular interest since this spread reflects the annualized cost of the put option associated with the extendible feature. In cases where the coupon on the straight 10-year bond is significantly less than on the extendible (which frequently occurs), an adjusted yield spread is also calculated. In essence, the adjustment attempts to raise the actual yield on the straight bond to that level which would prevail if both coupons were the same.

Consider first the yield comparisons with the straight 10-year bond. At the dates of issue of the extendibles, the actual yields on the straight 10-year bonds exceeded those on the extendibles by between one and 90 basis points. The median spread was 18 basis points and the average spread was 32 basis points. When an adjustment is made for coupon

*For the 18's of '86-'91, no "comparable" straight bonds exist. Therefore, worksheets are constructed for only 12 of the 13 extendible bonds identified in Table 1.

effects, the range is from 11 to 90 basis points, and the median and average spreads are 43 and 41 basis points, respectively.

Although the matching of the extendible and straight bonds is not perfect, and invariably involves comparing the yield on a new and a seasoned bond, the results are useful. Based on the coupon-adjusted data, the average amortized cost of the put option is 41 basis points. Further, this amortized cost varied substantially over time. This is to be expected, since the price of the put option - and hence its amortized cost - will depend both upon the term structure and on the degree of uncertainty regarding the future level of interest rates (Appendix C).

If symmetry prevails, so that the price of the put option is the same regardless of who owns it, these results are suggestive of the range in yield spreads that are likely to prevail between the proposed extendible and straight 5-year bonds. Because all of the extendible bonds that now exist are extendible at the initial coupon rate, it is not possible to infer how much this yield spread would fall if the option to extend in the proposed extendibles were at a higher coupon rate.*

A 5-year bond, extendible for an additional 5 years at the option of the holder is equivalent to the holder's owning a straight 5-year bond plus a call option (Appendix A). For this reason, the yield on the straight 5-year bond must always exceed that on the extendible. By comparing the two, one can obtain an estimate of the amortized cost of this call option. Although the key to the Pesando-Turnbull proposal lies in estimating the amortized cost of a put option, this additional comparison is worth conducting. In general, it is somewhat easier to match the extendible bonds with straight 5-year bonds, and the average

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*In the past, there have been extendible Canada bonds in which, if the holder elects to extend, the coupon rate increases. Such was the case, for example, with the 9-9½'s of '80-'85 and the 7½-8¾'s of '79-'84. All such bonds have now expired, as the option to extend was not exercised.

quality of the data is somewhat better on this account. The excess yield on the straight bonds, again at the dates of issue of the extendible bonds, ranged from 17 to 63 basis points, with a median of 43 basis points and an average of 41 basis points (Table 2). These data suggest that the amortized cost of the call option is typically less than 50 basis points, and can vary over time as market conditions change.

There is a way of looking at these results which is potentially quite useful in the context of the Pesando-Turnbull proposal. If (1) a futures price is an unbiased estimate of a subsequent spot price and (2) the distribution of prices is symmetric around this futures price, then the price of a call option with a striking price equal to the futures price must equal the price of a put option with this same striking price. If the term structure is flat and if the pure expectations theory holds, then the forward rate on 5-year bonds is the same as the spot rate, and represents an unbiased estimate of the subsequent spot rate. The prior result thus holds, and a put and a call option with a striking price equal to the current interest rate must have the same price. The median value of the call option, to the extent that these assumptions are met, thus conveys information about the median value of the put option. Interestingly, the average amortized cost of the put option (after correction for coupon effects) is 41 basis points, exactly equal to the average amortized cost of the call option. Although this exact result must be regarded as a coincidence, the suggestion is that a median value of 50 basis points for the amortized cost of the put option may be a reasonable "ball park" estimate.*

*Note, however, that if the term structure is flat - as has occasionally occurred in the past few years - one cannot in general conclude that expectations are that interest rates will remain at their present levels. Suppose, for example, that there is a substantial risk premium embedded in long-term interest rates. Then, even if the term structure is flat, the expectation may be that interest rates will fall sharply in the future. (If so, this could explain why the value of the put option identified by comparing the extendible with straight 10-year bonds often appears to be quite low.)

The range in yield spreads noted in the preceding paragraphs reflects the variations in the ex ante value (or, more precisely, its annual cost) of the put or call options. Like any option, the ex post changes in value will reflect - in the main - the movement in the price of the underlying security. From this perspective, it is useful to monitor the yield spread over time between a matched pair of extendible and straight bonds.

Consider then the behavior between 15 June 1982 and 20 April 1983 in the spread between the yield on the 15's of '87-'92 and the 15's of '92 (Table 3). When the extendible traded at par (9 August 1982), the yield on the straight bond exceeded that on the extendible by 50.1 basis points. This is a precise estimate of the annual cost of the put option at this date. When the extendible bond is selling at a discount in June and early July of 1982, the yield spread is larger and occasionally exceeds 100 basis points. In effect, this is due to the increased likelihood that the put option will be exercised since it is already "in the money". If interest rates were to remain unchanged from levels at which the extendible bond is trading at a discount, then the holder would exercise the put option and the bond would be redeemed after 5 years. By April of 1983, the extendible bond was trading well above par. Not surprisingly, the spread between the yield on the straight and the extendible bonds had fallen sharply, to the area of 20 basis points. This is due to the fact that, with yields to maturity of around 12%, the option to put the 15's of '87-'92 back to the Government at an interest rate of 15% has far less value. There remains, of course, the possibility that interest rates will rise sharply before 1987, which implies that the

option retains some value. But, in the jargon of the bond traders, the extendible is trading (almost) as if it were a straight 10-year bond.

Although our focus is on yield spreads because of the mechanics of the Pesando-Turnbull proposal for mortgage rate insurance, it is important to emphasize the economic significanct of market prices per se. In Table 3, note that the spread between the prices of the two bonds is quite small ($2.00 on 20 April 1983) when both are trading substantially above par. Since the two bonds have identical coupons, the difference in their prices (abstracting from possible liquidity effects) is equal to the price of the put option. For the reason cited previously, this price is small when interest rates are far beneath the striking price (15%) of the option. By contrast, the price of the extendible bond exceeds that of the straight bond by more than $4.00 during late June and early July of 1982. At this time, the extendible bond was trading beneath par, the put option was "in the money", and its price exceeded $4.00 (or $40.00 for each $1,000.00 of par value).*

As a second example, the 14¼'s of '87-'92 are compared in Table 3A to the 15's of '92. In August of 1982, when both bonds were trading slightly

*Although the interpretation of yield spreads in the present example is straightforward, so that focusing on prices is less necessary, this need not always be the case. On 20 April 1983, the 14¼'s of '87-'92 were selling at 115.375, while the 14⅜'s of '87 were selling at 112.75. In spite of the fact that the coupon on the extendible bond is lower, its price was higher. In this comparison, the extendible bond provides the holder with a call option, which is "in the money" when both bonds are trading above par. The higher price of the extendible reflects the value of this option. By contrast, however, the calculated yield to maturity is 10.81% for the straight bond and 11.52% for the extendible! This is due to the fact that the capital loss on the straight bond (which will be redeemed at par) is amortized over a 5-year period, while - following the standard convention - the capital loss on the extendible bond is amortized over 10 years. The convention of treating an extendible bond as if it were a straight 10-year bond if it is trading above par (and as if it were a straight 5-year bond if it is trading beneath par) is nonetheless a simplification. In the present context, it leads to the anomalous result that the calculated yield to maturity on the extendible bond exceeds that of the straight bond, in spite of the fact that the call option associated with the extendible bond is "in the money".

above par, the yield on the straight bond exceeded that on the extendible by as much as 87 basis points. In spite of the fact that the coupon on the extendible bond was less, its price frequently exceeded the price of the straight bond. This, of course, reflects the value of its associated put option. By April of 1983, however, the yield spread had fallen to about 40 basis points, and the price of the straight bond exceeded that of the extendible. In effect, the value of the put option had become much smaller, given the sharp drop in interest rates between August 1982 and April 1983, and the fact that the straight bond had a higher coupon was sufficient for it to command a higher price.

(ii) Retractables and Callables

A 5-year bond extendible for 5 years at the option of the issuer is equivalent to a straight 10-year bond less the value of a call option. The call option is, of course, held by the issuer. Because of the concern that the market may price an option differently if it is held by the issuer rather than the holder, it is potentially instructive to see how the market has priced callable versus otherwise identical straight bonds.

Unfortunately, it is not easy to obtain evidence on how the market prices this call option. Government of Canada bonds denominated in Canadian dollars have no call provisions.* Until the past 2 or 3 years, the standard format for both provincial and corporate bonds in Canada was a term to maturity of 20 or 25 years with 15 years of call protection.

*The Government has issued $U.S. denominated bonds with call provisions, including the 8⅜'s of '98 and the 9¼'s of '98. Unfortunately, there are no $U.S. denominated Government of Canada bonds of a comparable maturity that are presently outstanding.

Although there exist outstanding bonds with 10 years left to maturity, they are trading at such large discounts that the call provisions are not very relevant. At present, provincial bonds continue to be issued with this standard format, while corporates are typically issued with a term to maturity of 15 years and no call protection. As a consequence, it is not possible to use Canadian dollar bonds to estimate the market price (or, as before, the amortized cost) of the relevant call option. Further, one should note that call provision typically enable the issuer to call the bond at any time after a specified date. They thus represent "American" options. By contrast, the option contained in the bonds which are extendible at the option of the issuer can only be exercised at a single date, and thus are "European". For this reason, information obtained from data on callable bonds can be of only limited value in the present context, even if it were more readily available.

To provide some evidence on this issue, we looked to the U.S. bond market for signals as to how call options have been priced. Even here, however, the tendency for indentures to be standardized made it difficult to "pair" otherwise identical bonds that differed in their call provisions. We did, for example, compare the Bell Canada 14¼'s of June '91, which are callable at par in June '88, with the World Bank 15's of December '88, which have no call provisions. When the Bell Canada's were trading at par on 13 August 1982, their yield exceeded that on the World Bank's by 29 basis points. In general, however, there is no straightforward way to compare this or other call options to see if their price is symmetric regardless of who holds them. As noted previously, a 5-year bond extendible for 5 years at the holder's option is equivalent

to the holder owning a straight 5-year bond plus a call option. The ideal would be to calculate the excess yield on a 10-year bond callable at par after 5 years relative to a straight 10-year bond. Then, at the same point in time, calculate the excess yield on a straight 5-year bond relative to a 5-year bond extendible at the holder's option for an additional 5 years. If symmetry prevails, these excess yields should be approximately the same. Unfortunately, such comparisons are simply not possible.

Finally, we report in Appendix D the generally unsuccessful attempts to extract information from pairing retractable provincial and corporate bonds with their straight counterparts.

3. Survey of Opinion of Underwriters and Investment Managers

(1) The Analytics Reviewed

By invoking the twin assumptions of (1) the pure expectations theory and (2) that expectations are held with certainty, it is straightforward to show how the yield on the proposed extendible relative to a straight bond will depend upon the term structure of interest rates. This exercise is conducted in Appendix C, and the message can be stated succinctly. The sharper is the rise in yields with term to maturity at the date the extendible bond is issued, cet. par. the higher will be the excess of the yield on the extendible relative to the straight bond. If the term structure is positively sloped, then - under the pure expectations theory - the 5-year rate expected to prevail 5 years in the future exceeds the current 5-year rate. This increases the likelihood that a bond extendible at the issuer's option at the initial 5-year rate will in fact

be extended, and hence raises the excess of its yield over that on a straight 5-year bond. Equivalently, the value of the put option held by the issuer at the date of issue is higher than it would be if (say) the term structure were flat, and the holder of the proposed extendible bond must be compensated accordingly. As also noted in Appendix C, the value of the put option - and thus the excess yield - will be larger for any given term structure, the greater is the uncertainty about the future level of interest rates.

The point of the preceding discussion is that one must anticipate that the excess yield on the proposed extendible versus a straight bond can and will vary over time, and in a predictable manner. If the degree of uncertainty regarding the future level of interest rates rises, then cet. par. the excess yield will increase. In turn, this will signal that the market price of the mortgage rate insurance has also increased. The final message is that any attempt to solicit information at a given point in time regarding the likely market response to the proposed bonds will invariably be influenced by market conditions at that time.

The holder of the proposed extendible bond may be viewed as holding either (1) a straight 5-year bond less a put option or (2) a straight 10-year bond less a call option (Appendix A). As a result, we can conclude that the yield on the proposed extendible at its date of issue (i.e. at par) must exceed both the straight 5-year and the straight 10-year rates. This is an important analytical result. As of 15 April 1983, for example, the 11's of December '87 yielded 10.40%, while the 11.25's of February '93 yielded 11.05%. If the former is used as the 5-year rate and the latter as the 10-year rate, we can conclude

that a 5-year bond extendible for 5 years at the issuer's option and at the initial coupon rate of 10.40% must be priced to yield at least 11.05%.* The excess yield must therefore be at least 65 basis points (i.e. 11.05% - 10.40%). This is a key analytical result, since it allows us to impose a lower bound on the excess yield. If the term structure is rising, the yield on the proposed 5-year extendible bond must exceed the yield on the straight 5-year bond by at least the excess of the yield on the straight 10-year bond over the yield on the straight 5-year bond.

Finally, as discussed repeatedly in this report, the existing extendible and retractable bonds can be extended or retracted at the option of the holder. If symmetry prevails, then the price of the corresponding options should be identical regardless of whether they are held by the issuer or the holder. Empirically, however, this remains as yet an unanswered question. There is one result from option theory which is worthy of note, however. If agents indeed form risk-free hedge portfolios, or if we can view the market as if they did, then symmetry must prevail. The option must have the same price regardless of whether the issuer or the holder owns it. Since modern models of option pricing are grounded in the presumption that options are priced on the basis of risk-free hedge portfolios, the a priori case for symmetry merits note on this account.

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*Because the coupon on the 11¼'s is higher this bond will trade at a slightly higher yield due to tax effects. As a result, there is a slight upward bias to the estimated excess yield.

(2) A Rule of Thumb to Estimate the Amortized Cost of the Put Option at Each Point in Time

In Table 2, we presented the (coupon-adjusted) spreads between the yield on straight 10-year bonds and 5-year bonds extendible for an additional 5 years at the holder's option. These spreads represent estimates of the amortized cost of the put option held by the Government of Canada if it were to issue 5-year bonds extendible for an additional 5 years at its option, if symmetry prevails. We have defined above an analytical rule which places a lower bound on the amortized cost of this put option at each point in time.

In Table 4, we present (i) the actual yield spread (corresponding to the amortized cost of the put option) and (ii) the theoretical lower bound and (iii) the excess of the former over the latter. In order to estimate (ii), we chose at each point in time a straight 5-year and a straight 10-year bond with comparable coupon rates.* The mean excess of the actual amortized cost over the lower bound is 31 basis points. This result suggests the following "rule of thumb":

1. CALCULATE THE LOWER BOUND TO THE YIELD SPREAD, which is the maximum of (zero, the excess of the yield on a straight 10-year relative to a straight 5-year bond)

2. ADD 31 BASIS POINTS TO THIS LOWER BOUND

Since the excess of the actual yield spread over its lower bound does vary over time, this rule of thumb is only approximate. Nonetheless, does capture the crucial role of the term structure as an input into the

*On 25 August 1982, for example, we used the $14\frac{3}{4}$'s of June '87 as the straight 5-year bond, and the 15's of June '92 as the straight 10-year bond.

amortized cost of the put option, and uses the actual data on this amortized cost in an insightful way. This rule of thumb does, of course, require the assumption of symmetry, and this caveat should be borne in mind.

(3) Survey of Underwriters and Fund Managers

In order to obtain input from the investment community regarding the pricing and reception of the proposed extendible bonds, we circulated a questionnaire (Appendix E) to selected individuals.

The first respondent was an underwriter (Dominion Securities Ames), and the principal contact was Fred Blain, Vice-President and Director in charge of the bond department. Blain estimated that a 5-year bond, extendible for 5 additional years at the option of the Government at the initial coupon rate, would <u>today</u> command a yield premium of 75-100 basis points over a straight 5-year bond. At the time that Blain made this estimate, 10-year Canada's were trading at about 10.75%, while 5-year Canada's were trading at about 10.25%. Our "rule of thumb" would thus imply that the yield premium would be 50 plus 31 or 81 basis points, which is consistent with the estimate provided by Dominion Securities Ames.*

Dominion Securities Ames talked to three major accounts (Sun Life, Mutual Life and Beutel Goodman), whom they thought would be the key to the success of a potential offering to institutions. Their response was positive, but not enthusiastic. Mr. Blain's judgement was that an issue could be placed under current market conditions at a rate 1/4 to 1/2 point

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*Indirectly, we can thus infer that the respondents were coming down in favor of symmetry, given that symmetry is implicit in our "rule of thumb".

over the rate for a straight 10-year bond.* It should be noted, however, that while the institutions felt that this would be a fair pricing, they were not sure they would want to buy as a matter of principle.

With no similar liability or complementary asset against which investors could match the proposed bonds and so hedge themselves, the investors were being asked to fly directly in the face of the basic principle which has guided their investment policy over the past several years: matching the term structure of asset and liability portfolios in order to minimize risk. They evidently thought the proposal over carefully but were uncomfortable with it. Mr. Blain commented that it took the accounts an unusually long time and a considerable amount of prodding to come back with their opinions. While the eventual response was guardedly positive, therefore, it is likely that an instrument of this type would only attract a broad range of investors when the tone of the market is positive, as it is currently. Under adverse conditions, even with a larger premium, many potential investors could be expected to pass regardless of rate.

The accounts preferred to treat the instrument as a 10-year bond with a borrower call in five years, rather than as a 5-year issue with a borrower put for another 5 years. The reason is basically cosmetic: if the option were a put and it was exercised, the investor would have to enter the new bond in his portfolio at what would be by definition unfavourable terms. There would be far less risk of criticism if the

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*Again, since 10-year rates now exceed 5-year rates, this result is consistent with our "rule of thumb", which is (when the 10-year rate exceeds the 5-year rate) the 10-year rate plus 31 basis points.

option were a call: the bond would look good against other 10-year issues of similar vintage, and would simply disappear from the portfolio if the call were exercised. Callable bonds are also more familiar to the market, being fairly common in the United States. Indeed, Mr. Blain commented that a corporate borrower was right now exploring the possibility of issuing a 10-year bond callable after 6 years in the Canadian market. (These bonds are of course callable at any time after the call protection period is up, but are generally not called unless there is a large advantage to doing so.)

The accounts accepted readily the proposition that the yield premium would be less if the extendible feature were at a higher coupon rate. They were not, however, ready to speculate on how much. It would appear to require a specific issue to get the accounts to be more forthcoming on this matter.

Michelle Head, fixed income portfolio manager for Imperial Oil Limited, also responded to our questionnaire. As with Dominion Securities Ames, the respondent chose not to fill out the questionnaire, but to offer general comments. The fixed income analysts at Imperial Oil Limited estimated, on 1 May 1983, that a 5-year bond extendible for 5 years at the Government's option would trade at a premium of about 50 basis points over the straight 10-year rate. As of 1 May 1988, the yield on a straight 5-year Canada bond equalled (approx.) 10.00%, while the yield on a straight 10-year Canada bond equalled (approx.) 10.75%. The estimate of a 50 basis point premium over the 10-year rate thus implies a 125 basis point spread over the 5-year rate. Our "rule of thumb", as of 1 May 1983, would imply a yield spread of 75 plus 31 equals 106 basis points, or somewhat less than this estimate of 125 basis points. What

also merits emphasis is that both respondents (1) chose to price the proposed extendible off the straight 10-year rate and (2) preferred to view the extendible as a straight 10-year bond less a call option held by the issuer.

The fixed income analysts at Imperial Oil also drew attention to the fact that the yield premium required on the proposed extendible would vary over time, reflecting both the prevailing term structure and the volatility of interest rates. Both points are, of course, stressed earlier in this report. Attention was also drawn to the probable importance of the size of the issue, and the observation made that the 5 plus 5 extendible "has some appeal for participants".

Finally, the fact that both respondents chose to price the extendible off the straight 10-year bond merits emphasis. They are then pricing a call option, whose annual yield equivalent is estimated to be 25-50 and 50 basis points, respectively. Significantly, our estimates of the amortized cost of the call option (obtained by comparing the outstanding extendibles with a straight 5-year bond) were in the vicinity of 40 basis points. From this perspective, again, there seems to be basic similarity between the view of those in the investment community and the signals that can be extracted from existing debt instruments. (This, too, should not be surprising!)

Table 1

Extendable Bonds Issued by the Government of Canada

Coupon	Maturity Date	Amount Outstanding ($ millions)	Issue Date
13¾	'85-'90	850	Mar. 31/80
13	'85-'90	1,800	May 1/80 (and Dec. 1 & Dec. 22)
12½	'86-'91	725	Feb. 1/81
12½	'84-'90	775	Oct. 1/80
13¾	'84-'89	450	Mar. 1/81
13¼	'85-'90	600	Mar. 31/81
14½	'86-'91	500	May 1/81
15¼	'86-'93	900	June 1/81 & July 31/81
14¾	'86-'93	450	July 1/81
18	'86-'91	400	Oct. 15/81
15½	'87-'92	650/1250	Feb. 1/82 & July 1/82
15	'87-'92	400	July 1/82
14¼	'87-'92	650	Sept. 1/82

Table 2

Summary of Excess of Yields on Straight 5-Year and 10-Year Bonds
Relative to Extendable Government of Canada Bonds

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1. Excess of Yield on Straight 10-Year Bonds, in Basis Points

Actual Value	Range:	Median:	Average:
90, 86, 51, 42, 17, 14 15, 1, 15, 6, 18, 27	1-90	18	32
Coupon (tax) - Adjusted Values	**Range:**	**Median:**	**Average:**
90, 86, 74, 51, 17, 14 22, 11, 35, 22, 36, 39	11-90	36	41

2. Excess of Yield on Straight 5-Year Bonds, in Basis Points

Actual Values	Range:	Median:	Average:
43, 45, 63, 17, 31, 17 20, 25, 45, 52, 63, 27	17-63	43	41

Source: Worksheets contained in Appendix B.

Table 5		15.000 % 06/01/92		15.000 % 07/01/92		YIELD SPREAD	
		BID	YIELD	BID	YIELD	PRICE	YIELD
LATEST	04/20/83	116.875	11.906	118.875	11.596	2.000	-31.0
AVERAGE	216 DAYS	109.095	13.349	111.047	12.999	1.952	-34.9
BEST	03/14/83	116.875	11.920	117.500	11.833	0.625	-8.7
WORST	07/02/82	92.250	16.612	97.750	15.447	5.500	-116.5

((B)EG/GRAPH/REV/DETAILS(OLD)/$/Y/STATS../?/) : D

DETAILS

DATE	CAN15/92 BID	CAN15/92 YLD	VS	CAN15/87X BID	CAN15/87X YLD	SPREAD PRICE	SPREAD YLD
04/20/83	116.875	11.906		118.875	11.596	2.000	-31.0
04/19/83	116.875	11.906		118.875	11.596	2.000	-31.0
04/18/83	117.375	11.825		118.875	11.596	1.500	-22.9
04/15/83	117.250	11.847		118.750	11.618	1.500	-22.9
04/14/83	117.250	11.847		119.625	11.638	1.375	-20.9
04/13/83	116.875	11.909		118.250	11.699	1.375	-21.0
04/12/83	116.875	11.909		118.250	11.699	1.375	-21.0
04/11/83	117.000	11.889		118.250	11.699	1.250	-19.0
04/08/83	116.750	11.932		117.875	11.762	1.125	-17.0
04/07/83	116.750	11.932		117.875	11.762	1.125	-17.0
04/06/83	116.750	11.932		117.875	11.762	1.125	-17.0
04/05/83	116.750	11.932		117.875	11.762	1.125	-17.0
04/04/83	116.375	11.994		117.750	11.785	1.375	-20.9
03/31/83	116.500	11.975		117.750	11.785	1.250	-19.0
03/30/83	116.500	11.975		117.750	11.785	1.250	-19.0
03/29/83	116.500	11.975		117.750	11.785	1.250	-19.0
03/28/83	116.375	11.996		117.500	11.826	1.125	-17.0
03/25/83	116.625	11.955		117.500	11.826	0.875	-12.9
03/24/83	117.000	11.896		117.750	11.788	0.750	-10.8
03/23/83	116.750	11.937		117.500	11.828	0.750	-10.9
03/22/83	116.250	12.019		117.250	11.869	1.000	-15.0
03/21/83	116.500	11.980		117.375	11.851	0.875	-12.9
03/18/83	116.500	11.980		117.375	11.851	0.875	-12.9
03/17/83	116.625	11.960		117.625	11.812	1.000	-14.8
03/16/83	116.750	11.940		117.625	11.812	0.875	-12.8
03/15/83	116.875	11.920		117.625	11.812	0.750	-10.8
03/14/83	116.875	11.920		117.500	11.833	0.625	-8.7
03/11/83	116.375	12.003		117.500	11.834	1.125	-16.9
03/10/83	116.375	12.003		117.750	11.795	1.375	-20.8
03/09/83	116.500	11.983		117.750	11.795	1.250	-18.8
03/08/83	116.125	12.045		117.625	11.816	1.500	-22.0
03/07/83	116.250	12.025		117.750	11.796	1.500	-22.9
03/04/83	116.500	11.986		117.750	11.798	1.250	-18.8
03/03/83	116.625	11.966		117.875	11.779	1.250	-18.7
03/02/83	116.500	11.986		117.875	11.779	1.375	-20.8
03/01/83	116.500	11.987		117.750	11.799	1.250	-18.8
02/28/83	116.500	11.989		117.750	11.801	1.250	-18.7
02/25/83	116.500	11.989		117.750	11.801	1.250	-18.7
02/24/83	116.000	12.071		117.500	11.842	1.500	-22.9

02/22/83	116.500	11.990	118.000	11.763	1.500	-22.7
02/21/83	116.625	11.971	118.000	11.765	1.375	-20.6
02/18/83	116.625	11.971	118.000	11.765	1.375	-20.6
02/17/83	116.500	11.992	117.750	11.806	1.250	-18.7
02/16/83	116.250	12.033	117.500	11.846	1.250	-18.7
02/15/83	116.125	12.054	117.375	11.867	1.250	-19.8
02/14/83	116.500	11.994	117.375	11.867	0.875	-12.6
02/11/83	115.750	12.117	116.875	11.949	1.125	-16.8
02/10/83	115.500	12.159	116.500	12.010	1.000	-14.9
02/09/83	114.750	12.283	116.000	12.092	1.250	-19.2
02/08/83	114.625	12.305	116.000	12.092	1.375	-21.2
02/07/83	114.500	12.326	116.000	12.093	1.500	-23.3
02/04/83	114.250	12.369	116.000	12.095	1.750	-27.4
02/03/83	114.250	12.369	116.000	12.095	1.750	-27.4
02/02/83	114.125	12.391	115.750	12.136	1.625	-25.4
02/01/83	114.375	12.350	116.000	12.097	1.625	-25.3
01/31/83	114.375	12.350	116.000	12.097	1.625	-25.3
01/28/83	114.875	12.268	116.500	12.017	1.625	-25.1
01/27/83	114.875	12.268	116.500	12.018	1.625	-25.1
01/26/83	115.000	12.248	116.500	12.018	1.500	-23.0
01/25/83	115.000	12.249	116.500	12.019	1.500	-23.0
01/24/83	115.250	12.209	116.250	12.060	1.000	-14.8
01/21/83	115.500	12.168	116.500	12.021	1.000	-14.7
01/20/83	116.250	12.046	117.125	11.921	0.875	-12.5
01/19/83	116.125	12.067	117.125	11.922	1.000	-14.5
01/18/83	115.625	12.149	117.125	11.922	1.500	-22.7
01/17/83	116.375	12.027	117.750	11.823	1.375	-20.4
01/14/83	116.375	12.029	117.750	11.825	1.375	-20.4
01/13/83	116.500	12.009	118.000	11.784	1.500	-22.3
01/12/83	116.250	12.050	117.750	11.826	1.500	-22.4
01/11/83	116.250	12.051	117.750	11.827	1.500	-22.4
01/10/83	116.250	12.053	117.625	11.849	1.375	-20.4
01/07/83	116.250	12.053	117.625	11.849	1.375	-20.4
01/06/83	116.000	12.094	117.375	11.889	1.375	-20.5
01/05/83	116.250	12.054	117.250	11.910	1.000	-14.4
01/04/83	115.875	12.115	117.000	11.950	1.125	-16.5
12/31/82	116.000	12.097	116.750	11.993	0.750	-10.6
12/30/82	115.750	12.139	116.750	11.993	1.000	-14.5
12/29/82	115.750	12.139	116.500	12.034	0.750	-10.5
12/28/82	116.125	12.078	117.000	11.955	0.875	-12.4
12/24/82	115.750	12.142	116.750	11.997	1.000	-14.4
12/23/82	115.750	12.142	116.750	11.998	1.000	-14.4
12/22/82	115.750	12.143	116.750	11.998	1.000	-14.5
12/21/82	115.000	12.266	116.000	12.119	1.000	-14.7
12/20/82	114.500	12.349	115.500	12.201	1.000	-14.9
12/17/82	115.000	12.269	115.750	12.161	0.750	-10.8
12/16/82	115.000	12.269	115.750	12.161	0.750	-10.8
12/15/82	115.000	12.269	115.750	12.161	0.750	-10.8
12/14/82	114.500	12.352	115.500	12.202	1.000	-15.0
12/13/82	114.500	12.353	115.750	12.161	1.250	-19.1
12/10/82	114.000	12.437	115.250	12.243	1.250	-19.4
12/09/82	114.000	12.438	115.500	12.203	1.500	-23.5
12/08/82	113.750	12.480	115.500	12.203	1.750	-27.7
12/07/82	114.250	12.397	116.000	12.122	1.750	-27.5
12/06/82	114.750	12.315	116.500	12.042	1.750	-27.3
12/03/82	114.250	12.399	116.000	12.123	1.750	-27.4
12/02/82	113.250	12.567	115.000	12.284	1.750	-28.1
12/01/82	112.750	12.651	114.500	12.368	1.750	-29.3
11/30/82	112.750	12.652	114.250	12.409	1.500	-24.2
11/29/82	113.000	12.610	115.000	12.287	2.000	-32.4

11/26/82	114.000	12.445	115.500	12.206	1.500	-23.9
11/25/82	113.750	12.487	115.250	12.247	1.500	-24.0
11/24/82	113.250	12.571	115.250	12.247	2.000	-32.4
11/23/82	113.500	12.530	115.250	12.247	1.750	-28.2
11/22/82	113.500	12.530	115.250	12.248	1.750	-28.2
11/19/82	114.500	12.365	115.750	12.167	1.250	-19.7
11/18/82	114.250	12.404	115.500	12.208	1.250	-19.9
11/17/82	113.750	12.489	115.000	12.289	1.250	-20.0
11/16/82	113.250	12.573	114.500	12.371	1.250	-20.1
11/15/82	113.500	12.531	115.000	12.290	1.500	-24.1
11/12/82	114.000	12.448	116.125	12.109	2.125	-34.0
11/11/82	114.000	12.449	116.125	12.109	2.125	-33.9
11/10/82	114.500	12.366	116.250	12.089	1.750	-27.7
11/09/82	114.500	12.366	116.000	12.130	1.500	-23.7
11/08/82	114.000	12.449	115.500	12.211	1.500	-23.9
11/05/82	113.750	12.491	115.500	12.212	1.750	-29.0
11/04/82	114.000	12.450	115.500	12.212	1.500	-23.8
11/03/82	114.000	12.450	115.500	12.212	1.500	-23.8
11/02/82	113.750	12.492	115.250	12.253	1.500	-23.9
11/01/82	113.250	12.575	114.000	12.457	0.750	-11.8
10/29/82	112.000	12.784	113.250	12.582	1.250	-20.4
10/28/82	111.000	12.957	112.750	12.666	1.750	-29.1
10/27/82	110.000	13.129	112.250	12.750	2.250	-38.0
10/26/82	110.000	13.130	112.250	12.750	2.250	-38.0
10/25/82	110.000	13.130	112.250	12.750	2.250	-38.0
10/22/82	110.500	13.043	113.500	12.542	3.000	-50.1
10/21/82	110.500	13.043	112.500	12.709	2.000	-33.5
10/20/82	110.250	13.087	112.000	12.709	2.250	-37.8
10/19/82	110.500	13.046	112.500	12.709	2.000	-33.4
10/18/82	110.500	13.044	112.500	12.710	2.000	-33.4
10/15/82	110.000	13.131	111.750	12.837	1.750	-29.4
10/14/82	110.250	13.088	112.000	12.795	1.750	-29.3
10/13/82	110.500	13.045	112.000	12.795	1.500	-25.0
10/12/82	110.000	13.131	111.750	12.837	1.750	-29.4
10/08/82	109.250	13.262	111.000	12.965	1.750	-29.7
10/07/82	109.000	13.306	110.750	13.008	1.750	-29.8
10/06/82	108.000	13.483	110.000	13.137	2.000	-34.4
10/05/82	107.250	13.618	109.250	13.267	2.000	-35.0
10/04/82	106.500	13.753	109.000	13.311	2.500	-44.2
10/01/82	107.000	13.663	109.000	13.312	2.000	-35.1
09/30/82	106.500	13.753	109.000	13.312	2.500	-44.1
09/29/82	106.500	13.754	109.000	13.313	2.500	-44.1
09/28/82	106.500	13.754	109.000	13.313	2.500	-44.1
09/27/82	106.500	13.754	109.000	13.313	2.500	-44.1
09/24/82	106.500	13.754	109.250	13.270	2.750	-48.4
09/23/82	107.000	13.664	110.000	13.140	3.000	-52.3
09/22/82	106.750	13.709	109.500	13.227	2.750	-48.2
09/21/82	106.750	13.709	109.000	13.314	2.250	-39.5
09/20/82	105.500	13.937	107.500	13.579	2.000	-35.8
09/17/82	105.000	14.029	106.875	13.691	1.875	-33.8
09/16/82	104.500	14.122	106.500	13.759	2.000	-36.3
09/15/82	104.500	14.122	106.500	13.759	2.000	-36.3
09/14/82	104.250	14.169	106.750	13.714	2.500	-45.4
09/13/82	103.500	14.310	106.750	13.714	3.250	-59.5
09/10/82	103.250	14.357	105.750	13.896	2.500	-44.1
09/09/82	104.000	14.216	106.250	13.805	2.250	-41.0
09/08/82	104.500	14.122	106.875	13.693	2.375	-42.9
09/07/82	104.500	14.123	107.125	13.648	2.625	-47.4
09/03/82	104.500	14.123	106.875	13.693	2.375	-42.9
09/02/82	104.500	14.123	106.750	13.716	2.250	-40.6

09/02/82	104.500	14.123	106.750	13.718	2.250	
09/01/82	102.750	14.263	105.750	13.897	2.000	-34.6
08/31/82	103.500	14.310	105.000	14.035	1.500	-27.5
08/30/82	102.750	14.452	104.500	14.127	1.750	-32.5
08/27/82	102.500	14.500	104.250	14.174	1.750	-32.5
08/26/82	103.750	14.264	105.250	13.990	1.500	-27.4
08/25/82	103.750	14.264	105.250	13.990	1.500	-27.3
08/24/82	103.750	14.264	105.250	13.990	1.500	-27.3
08/23/82	102.500	14.500	104.000	14.221	1.500	-27.9
08/20/82	102.250	14.549	103.750	14.269	1.500	-28.0
08/19/82	102.000	14.597	103.500	14.316	1.500	-28.1
08/18/82	101.750	14.645	103.750	14.269	2.000	-37.6
08/17/82	101.000	14.790	103.500	14.316	2.500	-47.4
08/16/82	99.250	15.136	102.250	14.553	3.000	-58.3
08/13/82	98.750	15.236	102.250	14.553	3.500	-68.3
08/12/82	98.000	15.388	101.000	14.794	3.000	-59.5
08/11/82	97.500	15.491	100.250	14.940	2.750	-55.0
08/10/82	97.500	15.491	100.125	14.965	2.625	-52.6
08/09/82	97.500	15.491	100.000	14.990	2.500	-50.1
08/06/82	97.250	15.540	99.750	15.040	2.500	-50.2
08/05/82	98.000	15.388	100.125	14.966	2.125	-42.3
08/04/82	97.250	15.542	99.750	15.040	2.500	-50.2
08/03/82	97.000	15.594	99.500	15.090	2.500	-50.4
08/02/82	97.000	15.594	99.500	15.090	2.500	-50.4
07/30/82	96.250	15.750	98.875	15.215	2.625	-53.5
07/29/82	96.000	15.802	98.375	15.315	2.375	-48.6
07/28/82	95.750	15.854	98.250	15.341	2.500	-51.4
07/27/82	95.750	15.854	98.250	15.341	2.500	-51.3
07/26/82	95.250	15.960	98.000	15.392	2.750	-56.8
07/23/82	95.750	15.854	98.250	15.342	2.500	-51.3
07/22/82	95.500	15.907	98.500	15.291	3.000	-61.6
07/21/82	95.000	16.013	98.250	15.342	3.250	-67.1
07/20/82	95.500	15.907	98.375	15.317	2.875	-59.0
07/19/82	95.000	16.013	98.000	15.393	3.000	-62.0
07/16/82	94.500	16.120	98.000	15.393	3.500	-72.7
07/15/82	93.500	16.334	97.625	15.470	4.125	-84.7
07/14/82	93.500	16.336	97.500	15.495	4.000	-84.1
07/13/82	93.250	16.391	97.375	15.521	4.125	-87.0
07/12/82	93.250	16.391	97.625	15.470	4.375	-93.1
07/09/82	93.250	16.391	97.625	15.471	4.375	-92.0
07/08/82	93.625	16.328	97.125	15.573	4.500	-95.5
07/07/82	93.250	16.612	97.000	15.599	4.750	-101.2
07/06/82	93.250	16.612	97.000	15.599	4.750	-101.2
07/05/82	93.000	16.446	97.875	15.421	4.875	-102.5
07/02/82	93.250	16.612	97.750	15.447	5.500	-116.5
06/30/82	93.750	16.501	98.000	15.397	5.250	-110.4
06/29/82	93.750	16.501	97.375	15.524	4.625	-97.7
06/28/82	93.500	16.337	97.875	15.423	4.375	-91.4
06/25/82	94.000	16.229	98.000	15.398	4.000	-83.1
06/24/82	94.000	16.229	98.375	15.323	4.375	-90.6
06/23/82	94.000	16.229	98.375	15.323	4.375	-90.6
06/22/82	93.000	16.446	97.125	15.576	4.125	-87.0
06/21/82	93.000	16.446	96.875	15.627	3.875	-81.9
06/18/82	92.250	16.612	96.625	15.677	4.375	-93.5
06/17/82	93.875	16.256	97.250	15.549	3.375	-70.9
06/16/82	94.500	16.122	97.375	15.523	2.875	-59.9
06/15/82	95.125	15.989	98.125	15.370	3.000	-61.9

((B)EC/GRAPH/REV/DETAILS(OLD)/S/Y/STATS.../?/) : CAN12.5/6XB

((E)EC/GRAPH/REV/DETAILS(OLD)/$/Y/STATS ../?/) B

C

: AN15/92 CAN14 25% 42382

YIELD SPREAD ANALYSIS

Table 3A DAILY FROM 08/17/82 TO 04/25/83

		SELL CANADA 15 000 % 06/01/92		BUY EXTENDIBLE TO 14.250 % 09/01/92		YIELD SPREAD	
		BID	YIELD	OFFER	YIELD	PRICE	YIELD
LATEST	04/25/83	117 000	11 885	115 750	11 456	-1 250	-42 9
AVERAGE	175 DAYS	112 759	12 669	111 693	12 170	-1 094	-47 9
BEST	03/14/83	116 875	11.920	114 750	11 636	-2 125	-28 4
WORST	05/17/82	101 000	14 790	101 750	13 918	0 750	-87 2

((E)EC/GRAPH/REV/DETAILS(OLD)/$/Y/STATS ../?/) D

DETAILS

DATE	CAN15/92 BID	YLD	VS	CAN14 25% OFFER	YLD	SPREAD PRICE	YLD
04/25/83	117 000	11 885		115.750	11.456	-1 250	-42 9
04/22/83	117 000	11 885		115 750	11.456	-1 250	-42 9
04/21/83	116 750	11 926		115.500	11.497	-1 250	-42 9
04/20/83	116 875	11 906		115 625	11.478	-1 250	-42 9
04/19/83	116 875	11 906		115 625	11 478	-1 250	-42 9
04/18/83	117.375	11.815		115 750	11 459	-1 625	-35 7
04/15/83	117.250	11 847		115 625	11.480	-1 625	-36 7
04/14/83	117 250	11 847		115.500	11 500	-1 750	-34 7
04/13/83	116.875	11.909		115 125	11 561	-1 750	-34 8
04/12/83	116.875	11.909		115 125	11 561	-1.750	-34 8
04/11/83	117.000	11.839		115.125	11.562	-1 875	-32 7
04/08/83	116 750	11 932		114 875	11.605	-1 875	-32 7
04/07/83	116 750	11.932		114 875	11 605	-1 875	-32 7
04/06/83	116 750	11 932		114.875	11.605	-1.875	-32 7
04/05/83	116 750	11 932		114.875	11.605	-1 875	-32 7
04/04/83	116 375	11.991		114 750	11 628	-1 625	-36 5
03/31/83	116 500	11.975		114 750	11.628	-1.750	-34 7
03/30/83	116 500	11 975		114 750	11 628	-1 750	-34 7
03/29/83	116 500	11 975		114.750	11.628	-1 750	-34 7
03/28 83	116 375	11.996		114.750	11.629	-1.625	-36 7
03/25/83	116 625	11.955		114 750	11.630	-1 875	-32 5
03/24/83	117 000	11.896		115 000	11.591	-2.000	-30.5
03.23/83	116.750	11.937		114 750	11 631	-2 000	-30 5
03/22/83	116 250	12 019		114.375	11.692	-1.875	-32 7
03/21/83	116 500	11 980		114.500	11.674	-2 000	-30 5
03/18/83	116.500	11 980		114.500	11.674	-2 000	-30 5
03/17/83	116 625	11.960		114.750	11.636	-1 875	-32 4

03/16/83	116 750	11 960	114 750	11 636	-2 000	-31 4
03/15/83	116 875	11 920	114 750	11 636	-2 125	-28 4
03/14/83	116 875	11 920	114 750	11 636	-2 125	-28 4
03/11/83	116 375	12 003	114 750	11 638	-1 625	-36 5
03/10/83	116 375	12 003	115 000	11 599	-1 375	-40 5
03/09/83	116 500	11 983	115 000	11 599	-1 500	-38 5
03/08/83	116 125	12 045	115 000	11 599	-1 125	-44 4
03/07/83	116 250	12 025	115 250	11 560	-1 000	-46 5
03/04/83	116 500	11 984	115 500	11 500	-1 000	-48 4
03/03/83	116 625	11 966	115 625	11 503	-1 000	-46 3
03/02/83	116 500	11 984	115 625	11 503	-0 875	-48 3
03/01/83	116 500	11 987	115 500	11 524	-1 000	-46 3
02/28/83	116 500	11 999	115 500	11 524	-1 000	-47 4
02/25/83	116 500	11 999	115 500	11 524	-1 000	-44 4
02/24/83	116 000	12 071	115 250	11 564	-0 750	-50 6
02/23/83	115 875	12 092	115 000	11 604	-0 875	-49 7
02/22/83	116 500	11 990	115 500	11 525	-1 000	-47 5
02/21/83	116 625	11 971	115 500	11 526	-1 125	-44 5
02/18/83	116 625	11 971	115 500	11 526	-1 125	-44 5
02/17/83	116 500	11 990	115 375	11 546	-1 125	-44 6
02/16/83	116 250	12 032	115 125	11 586	-1 125	-44 7
02/15/83	116 125	12 054	115 000	11 606	-1 125	-44 5
02/14/83	116 500	11 994	115 001	11 606	-1 501	-39 7
02/11/83	115 750	12 117	114 500	11 687	-1 250	-43 0
02/10/83	115 500	12 159	114 250	11 727	-1 250	-43 2
02/09/83	114 750	12 283	113 500	11 849	-1 250	-43 5
02/08/83	114 625	12 305	113 500	11 849	-1 125	-45 6
02/07/83	114 500	12 326	113 500	11 849	-1 000	-47 7
02/04/83	114 250	12 369	113 500	11 850	-0 750	-51 9
02/03/83	114 250	12 369	113 375	11 870	-0 875	-49 9
02/02/83	114 125	12 391	113 000	11 931	-1 125	-45 9
02/01/83	114 375	12 350	113 375	11 971	-1 000	-37 9
01/31/83	114 375	12 350	113 250	11 891	-1 125	-45 9
01/28/83	114 875	12 269	113 750	11 811	-1 125	-45 7
01/27/83	114 875	12 269	113 750	11 811	-1 125	-45 7
01/26/83	115 000	12 249	114 000	11 771	-1 000	-47 7
01/25/83	115 000	12 249	114 250	11 731	-0 750	-51 7
01/24/83	115 250	12 208	113 750	11 812	-1 500	-39 6
01/21/83	115 500	12 168	114 000	11 772	-1 500	-39 6
01/20/83	116 250	12 046	114 750	11 653	-1 500	-39 4
01/19/83	116 125	12 067	114 625	11 673	-1 500	-39 4
01/18/83	115 625	12 149	114 625	11 673	-1 000	-47 6
01/17/83	116 375	12 027	115 000	11 614	-1 375	-41 4
01/14/83	116 375	12 029	115 000	11 615	-1 375	-41 4
01/13/83	116 500	12 009	115 000	11 615	-1 500	-39 4
01/12/83	116 250	12 050	115 125	11 595	-1 125	-45 5
01/11/83	116 250	12 051	115 000	11 616	-1 250	-43 5
01/10/83	116 250	12 053	115 000	11 617	-1 250	-43 6
01/07/83	116 250	12 053	115 000	11 617	-1 250	-43 6
01/06/83	116 000	12 094	115 000	11 617	-1 000	-47 7
01/05/83	116 250	12 054	114 750	11 657	-1 500	-39 7
01/04/83	115 875	12 115	114 250	11 737	-1 625	-37 9
12/31/82	116 000	12 097	114 250	11 738	-1 750	-35 9
12/30/82	115 750	12 135	114 000	11 779	-1 750	-34 6
12/29/82	115 750	12 139	114 000	11 779	-1 750	-34 8
12/28/82	116 125	12 078	114 250	11 739	-1 875	-33 9
12/24/82	115 750	12 142	114 000	11 780	-1 750	-34 8
12/23/82	115 750	12 142	114 000	11 780	-1 750	-34 8
12/22/82	115 750	12 140	114 000	11 781	-1 750	-34 8
12/21/82	115 000	12 264	113 000	11 942	-2 000	-32 9

12/20/82	114.500	12.349	112.500	12.023	-2.000	-31.6
12/17/82	115.000	12.268	112.000	11.943	-2.000	-31.5
12/16/82	115.000	12.269	113.000	11.944	-2.000	-32.6
12/15/82	115.000	12.269	113.000	11.944	-2.000	-32.6
12/14/82	114.500	12.352	112.500	12.025	-2.000	-32.7
12/13/82	114.500	12.353	113.750	11.985	-1.750	-34.9
12/10/82	114.000	12.437	112.250	12.047	-1.750	-37.1
12/09/82	114.000	12.438	112.500	12.027	-1.500	-41.1
12/08/82	113.750	12.480	112.500	12.027	-1.250	-45.2
12/07/82	114.250	12.397	113.000	11.944	-1.250	-45.1
12/06/82	114.750	12.315	114.000	11.786	-0.750	-50.9
12/03/82	114.250	12.399	113.000	11.947	-1.250	-45.2
12/02/82	113.250	12.567	112.000	12.110	-1.250	-45.7
12/01/82	112.750	12.651	111.500	12.191	-1.250	-45.9
11/30/82	112.750	12.652	111.250	12.234	-1.500	-41.9
11/29/82	113.000	12.610	111.500	12.193	-1.500	-41.7
11/24/82	114.000	12.445	112.250	12.071	-1.750	-37.4
11/23/82	113.750	12.487	112.000	12.113	-1.750	-37.4
11/22/82	113.250	12.571	112.000	12.113	-1.250	-45.8
11/22/82	113.500	12.530	112.250	12.072	-1.250	-45.8
11/19/82	113.500	12.530	112.250	12.072	-1.250	-45.8
11/18/82	114.500	12.365	112.750	11.992	-1.750	-37.1
11/17/82	114.250	12.406	113.000	12.033	-1.750	-37.3
11/16/82	113.750	12.489	113.000	12.113	-1.750	-37.4
11/15/82	113.250	12.573	111.500	12.197	-1.750	-37.4
11/12/82	113.500	12.531	112.250	12.075	-1.250	-45.5
11/11/82	114.000	12.448	113.125	11.934	-0.875	-51.4
11/10/82	114.000	12.449	113.125	11.935	-0.875	-51.4
11/09/82	114.500	12.366	113.250	11.915	-1.250	-45.1
11/08/82	114.500	12.366	112.750	11.996	-1.750	-37.1
11/05/82	114.000	12.449	112.750	11.996	-1.250	-45.3
11/04/82	113.750	12.491	112.750	11.997	-1.000	-45.4
11/03/82	114.000	12.450	112.750	11.99	-1.250	-45.2
11/02/82	113.750	12.492	[illegible]500	12.039	-1.250	-45.3
11/01/82	113.250	12.5[illegible]	111.500	12.202	-1.750	-37.3
10/29/82	112.[illegible]	12.786	111.000	12.285	-1.000	-50.1
10/28/82	[illegible]000	12.957	110.000	12.451	-1.000	-50.5
10/27	110.000	13.129	109.250	12.577	-0.750	-55.2
10 82	110.000	13.130	109.250	12.578	-0.750	-55.2
10/25/82	110.000	13.130	109.250	12.578	-0.750	-55.2
10/22/82	110.500	13.043	109.750	12.495	-0.750	-54.9
10/21/82	110.500	13.043	109.750	12.495	-0.750	-54.9
10/20/82	110.250	13.087	109.750	12.496	-0.500	-59.1
10/19/82	110.500	13.044	109.750	12.496	-0.750	-54.9
10/18/82	110.500	13.044	109.500	12.538	-1.000	-50.4
10/15/82	110.000	13.131	109.000	12.623	-1.000	-50.7
10/14/82	110.250	13.088	109.250	12.581	-1.000	-50.4
10/13/82	110.500	13.045	109.625	12.519	-0.875	-52.4
10/12/82	110.000	13.131	109.500	12.540	-0.500	-59.1
10/09/82	109.250	13.261	108.250	12.753	-1.000	-50.9
10/07/82	109.000	13.306	105.000	12.796	-1.000	-51.0
10/06/82	108.000	13.483	107.125	12.947	-0.875	-53.7
10/05/82	107.250	13.618	106.750	13.012	-0.500	-60.5
10/04/82	106.500	13.753	106.250	13.100	-0.250	-65.4
10/01/82	107.000	13.662	106.750	13.012	-0.250	-65.0
09/30/82	106.500	13.753	106.500	13.057	0.000	-69.6
09/29/82	106.500	13.754	106.500	13.057	0.000	-69.6
09/28/82	106.500	13.754	105.750	13.189	-0.750	-54.4

10/21/82	110 500	13.043	109.750	12.495	-0 750	-59 6
10/20/82	110.250	13 087	109.750	12 496	-0 500	-59 1
10/19/82	110.500	13.044	109.750	12.496	-0.750	-59 8
10/18/82	110.500	13.044	109 500	12 538	-1 000	-60 4
10/15/82	110 000	13 131	109.000	12.623	-1 000	-60 7
10/14/82	110.250	13 088	109.250	12 581	-1 000	-60 4
10/13/82	110 500	13 045	109.625	12.519	-0 875	-60 4
10/12/82	110 000	13 131	109.500	12.540	-0 500	-59 1
10/09/82	109 250	13 261	108 250	12 753	-1 000	-60 6
10/07/82	109 000	13 306	105 000	12 796	-1 000	-61 0
10/06/82	108 000	13 493	107.125	12.947	-0 875	-59 7
10/05/82	107 250	13 619	106 750	13.012	-0 500	-60 5
10/04/82	106 500	13 753	106 250	13 100	-0 250	-60 6
10/01/82	107 000	13 663	106 750	13.013	-0 250	-60 0
09/30/82	106 500	13.753	106 500	13.057	0 000	-60 4
09/29/82	106 500	13 754	106.500	13.057	0 000	-60 4
09/28/82	106 500	13 754	105.750	13 199	-0 750	-62 4
09/27/82	106 500	13 754	106 875	12 998	0 375	-61 1
09/24/82	106 500	13 754	107.000	12.972	0 500	-79 1
09/23/82	107 000	13 664	107.875	12 822	0 875	-84 2
09/22/82	106 750	13 709	107 125	12.951	0 375	-75 9
09/21/82	106 750	13 709	106 375	13 082	-0 375	-62 7
09/20/82	105 500	13 937	105 625	13 214	0 125	-72 3
09/17/82	105 000	14 029	104 875	13.347	-0 125	-68 3
09/16/82	104 500	14 122	104 625	13 392	0 125	-73 0
09/15/82	104 500	14 122	104 625	13.393	0 125	-73.9
09/14/82	104 250	14 169	104 875	13.348	0 625	-85 0
09/13/82	103 500	14 310	104 375	13.438	0 875	-87 2
09/10/82	103 250	14.357	103 500	13 597	0 250	-76 0
09/09/82	104 000	14.216	104 250	13.462	0.250	-75 4
09/08/82	104.500	14 122	104 875	13 350	0 375	-77 2
09/07/82	104 500	14 123	104 875	13.350	0 375	-77 2
09/03/82	104 500	14 123	104.750	13.374	0 250	-74 9
09/02/82	104.500	14 123	104.625	13 397	0 125	-71 6
09/01/82	103 750	14 263	103 500	13.599	-0 250	-66 4
08/31/82	103.500	14 310	102.875	13.713	-0 625	-69 7
08/30/82	103 750	14 452	102 375	13 805	-0 375	-64 7
08/27/82	102 500	14 500	102 250	13.827	-0 250	-67 3
08/26/82	103 750	14 264	103 375	13 621	-0 375	-64 3
08/25/82	103.750	14 264	103.250	13.644	-0 500	-62 0
08/24/82	103 750	14.264	103 200	13.653	-0 550	-61 1
08/23/82	102 500	14 500	101.875	13.896	-0.625	-60 4
08/20/82	102 250	14 549	101 750	13.919	-0 500	-63 0
08/19/82	102 000	14 597	101.375	13 988	-0 625	-60 8
08/18/82	101 750	14 645	101.875	13.895	0 125	-75 0
08/17/82	101.000	14 790	101.750	13.918	0 750	-87 2

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Table 4

Actual and Theoretical Lower Bounds on the Amortized Cost of the Put Option

Worksheet No.	(1) Actual Spread	(2) Lower Bound	(3) Excess of (1) over (2)
1	90	47	43
2	86	41	45
3	74	7	67
4	51	29	22
5	17	0	17
6	14	0	14
7	22	0	22
8	11	0	11
9	35	0	35
10	22	0	22
11	36	0	36
12	39	0	39

Average of Excess of (1) over (2): 31

Appendix A

EXTENDABLE, RETRACTABLE, AND CALLABLE BONDS

The purpose of this Appendix is to identify the different options attached to extendable, retractable, and callable bonds. For the types of extendable and retractable bonds that currently exist in North American financial markets, the bond holder has the option to extend or retract the maturity of the bond. For callable bonds, it is the bond issuer and not the bond holder that has the option to call the bonds. It will be shown that the identity of the party that has the ownership of the option is an important factor in the pricing of the debt instrument.

Extendable Bonds

Consider a bond where in n years time, the bond holder has the option to extend the maturity of the bond for m more years. If the option is not used, the bond matures at the end of year n. In purchasing an extendable bond, the bond holder buys a bond of maturity n years and an option to buy, in year n, a bond of maturity m years for a price of F dollars, where F is the initial face value of the bond.

The market value of the extendable at the end of year n is the maximum of the face value, F, of the bond and the market value of the bond if it is extended for m more years, B_m. This can be written in the form

$$\begin{cases} B_m & ; \quad B_m \geq F \\ F & ; \quad B_m < F \end{cases}$$

$$= \begin{cases} F & ; \quad B_m \geq F \\ F & ; \quad B_m < F \end{cases} + \begin{cases} B_m - F & ; \quad B_m \geq F \\ 0 & ; \quad B_m < F \end{cases}$$

The first term in the above expression is the face value of the bond if it is not extended; and the second term is the value of a call option. Hence, the market value of the extendable bond equals the market value of a non-extendable bond with the same coupon plus the market value of a call option. The call option provides the bond holder with the option to extend the maturity of the extendable bond for m more years.

For the purposes of the present study, it is useful to interpret the market value of the extendable bond in terms of a put option and not a call option. By purchasing the extendable bond, the bond holder is effectively purchasing a bond of maturity $(n+m)$ years plus a put option to sell the bond at year n for a price of F dollars. At year n the bond holder will sell the bond (exercise the put option) if the bond price, B_m, is less than F. The market value of the bond is thus

$$\begin{cases} B_m & ; \quad B_m \geq F \\ F & ; \quad B_m < F \end{cases}$$

$$= \begin{cases} B_m & ; \quad B_m \geq F \\ B_m & ; \quad B_m < F \end{cases} + \begin{cases} 0 & ; \quad B_m \geq F \\ F - B_m & ; \quad B_m < F \end{cases}$$

Hence the market value of the extendable bond equals the market value of a non-extendable bond of maturity $n+m$ years, plus the market value of a put option.

Retractable Bonds

Consider a bond with an initial maturity of n years. After t years, the bond holder has the option to retract the maturity of the bond to m years $(t+m<n)$. Let ${}_tB_\Gamma$ denote the market value at time t of a bond of maturity Γ. Hence at time t, the bond holder will decide to retract the maturity of the bond if the market value of a m year bond, ${}_tB_m$, is greater than the market value of the bond if its maturity is not retracted, ${}_tB_{n-t}$. The market value of the retractable bond at time t can be written in the form

$$\begin{cases} {}_tB_{n-t} & ; \quad {}_tB_{n-t} \geq {}_tB_m \\ {}_tB_m & ; \quad {}_tB_{n-t} < {}_tB_m \end{cases}$$

$$= \begin{cases} {}_tB_{n-t} & ; \quad {}_tB_{n-t} \geq {}_tB_m \\ {}_tB_{n-t} & ; \quad {}_tB_{n-t} < {}_tB_m \end{cases} + \begin{cases} 0 & ; \quad {}_tB_{n-t} \geq {}_tB_m \\ {}_tB_m - {}_tB_{n-t} & ; \quad {}_tB_{n-t} < {}_tB_m \end{cases}$$

The first term in the above expression is the market value of the bond if it is not retracted; and the second term is the value of a put option. Hence, the market value of a retractable bond equals the market value of a non-retractable bond with the same coupon and of maturity n years plus the market value of a put option. The put option provides the bond holder with the option to retract the maturity of the bond.

Callable Bonds

The bond issuer has an option to call the bonds. In the event of the bonds being called, the issuer buys back the bonds at a pre-determined price, the call price. Typically, the issuer can only call the bonds after a certain specified period. The call price may vary over time. Such details are given in the bond indenture.

Consider a bond which can be called after n years. Let CP_n represent the call price. If the bond has a maturity of m years remaining at the end of year n, and if the bond issuer follows a rational call policy, then the bond will be called at the end of year n, if the market value of the bond, B_m, is greater than the call price, CP_n. Thus the value of the bond at the end of year n is

$$\begin{cases} CP_n & ; \quad B_m \geq CP_n \\ B_m & ; \quad B_m < CP_n \end{cases}$$

$$= \begin{cases} B_m & ; \quad B_m \geq CP_n \\ B_m & ; \quad B_m < CP_n \end{cases} - \begin{cases} B_m - CP_n & ; \quad B_m \geq CP_n \\ 0 & ; \quad B_m < CP_m \end{cases}$$

Hence the market value of a bond maturity n years, callable after m years, is equal to the market value of a non-callable bond with the same coupon and maturity n years minus the market value of a call option. The call option allows the issuer to call the bonds.

For the purposes of the present study, it is useful to interpret the market value of the callable bond in terms of a put option and not a call option. By purchasing a callable bond, the bond holders effectively

own a non-callable bond of maturity n years and face value CP_n and they have sold a put option to the bond issuer. At the end of year n if the market value of a m year bond B_m is greater than CP_n, the bond issuer will not exercise the option. If B_m is less than CP_n, the bond issuer will exercise the put option. The market value of the bond is thus

$$\begin{cases} CP_n ; & B_m \geq CP_n \\ B_m ; & B_m < CP_n \end{cases}$$

$$= \begin{cases} CP_n ; & B_m \geq CP_n \\ CP_n ; & B_m < CP_n \end{cases} + \begin{cases} 0 & B_m \geq CP_n \\ CP_n - B_m ; & B_m < CP_n \end{cases}$$

The first term in the above expression is the payoff of a bond with face value CP_n that matures at the end of year n, and the second term is the payoff of a put option.

In the above analysis of callable bonds, it has been assumed that the bond can only be called at a particular point in time. In general, this is not the case, implying that such options will be American options and not European options.

A summary of results is given in Table A1.

Table A1

Summary of Results

RESULT 1

The market value of a 5-year bond extendible for 5 years equals the market value of a 10-year bond retractable for 5 years.

RESULT 2

Owner of Option	MV of 5 + 5 Extendible Bond Equals	MV of 10 - 5 Retractable Bond Equals
Borrower	$MV_s(5)$ - MV of Put	$MV_s(10)$ - MV of Call
Lender	$MV_s(5)$ + MV or Call	$MV_s(5)$ + MV of Call
	$MV_s(10)$ + MV of Put	$MV_s(10)$ + MV of Put

where $MV_s(T)$ represents the market value of a straight bond of maturity T years.

Appendix B

Worksheets on Extendible versus Straight 5-Year and 10-Year Bonds

#1

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 14¼'s 1 Sept. '87-'92
   Amount Outstanding: 650
   Issue Date: 1 Sept. '82

2. Straight Bond with Same Short Maturity: 14¾'s 1 June '87
   Amount Outstanding: 250
   Issue Date: 1 June '82

3. Straight Bond with Same Long Maturity: 15's 1 June '92
   Amount Outstanding: 200
   Issue Date: 1 June '82

4. Yields at Issue Date of Extendible Bond (25 August '82):

	Yield	Price	Yield Spread
Extendible	13.36	103.19	—
Straight "Short"	13.79	103.18	+0.43
Straight "Long"	14.26	103.75	+0.90

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
13.44	13.39	13.80	13.96

6. Relevant Notes (if any)

#2

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 15's 1 July '87-'92
   Amount Outstanding: 400
   Issue Date: 1 July 1982

2. Straight Bond with Same Short Maturity: $14\frac{3}{4}$'s 1 June '87
   Amount Outstanding: 250
   Issue Date: 1 June '82

3. Straight Bond with Same Long Maturity: 15's 1 June '92
   Amount Outstanding: 200
   Issue Date: 1 June '82

4. Yields at Issue Date of Extendible Bond (30 June '82):

	Yield	Price	Yield Spread
Extendible	15.53	98.19	—
Straight "Short"	15.98	95.87	+0.45
Straight "Long"	16.39	93.25	+0.86

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
16.24	15.85	15.90	16.03

6. Relevant Notes (if any)

#3

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 15½'s 1 February '87-'92
   Amount Outstanding: 650
   Issue Date: 1 February '82

2. Straight Bond with Same Short Maturity: 8¾'s 1 February '88
   Amount Outstanding: 125
   Issue Date: 1 February '78

3. Straight Bond with Same Long Maturity: 9½'s 15 June '94
   Amount Outstanding: 845
   Issue Date: 15 June '74 (and subsequent)

4. Yields at Issue Date of Extendible Bond (27 January '82):

	Yield	Price	Yield Spread
Extendible	15.43	100.25	—
Straight "Short"	16.06	72.50	+0.63
Straight "Long"	15.94	65.62	+0.51 (0.74)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
15.93	15.73	15.95	15.94

6. Relevant Notes (if any)

* Comparing 9¾'s of '00 with 15¾'s of '01, find yield spread of 23 basis points in favour of latter. If add 23 basis points to 0.51 to get a tax adjustment, obtain 0.74 as spread between straight "long" and extendible bond.

#4

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 13's, 1 May '85-'90
   Amount Outstanding: 1,800
   Issue Date: 1 May '80 and subsequently

2. Straight Bond with Same Short Maturity: 11¼'s 15 December '84
   Amount Outstanding: 700
   Issue Date: 15 December '79

3. Straight Bond with Same Long Maturity: 11¼'s 15 December '89
   Amount Outstanding: 325
   Issue Date: 15 December '79 and subsequently

4. Yields at Issue Date of Extendible Bond (30 April '80):

	Yield	Price	Yield Spread
Extendible	11.63	105.06	—
Straight "Short"	11.80	98.94	+0.17
Straight "Long"	12.05	95.50	+0.42 (+0.51)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
12.35	11.74	11.90	12.01

6. Relevant Notes (if any)

* If use "rule of thumb" that a 1% higher coupon is worth about 5 basis points, then yield spread rises to 51 basis points.

#5

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 12¼'s 1 October '84-'90
   Amount Outstanding: 775
   Issue Date: 1 October '80

2. Straight Bond with Same Short Maturity: 10¼'s 1 October '84
   Amount Outstanding: 300
   Issue Date: 1 October '79

3. Straight Bond with Same Long Maturity: 11¼'s 15 December '89
   Amount Outstanding: 325
   Issue Date: 15 December '79

4. Yields at Issue Date of Extendible Bond (24 September '80):

	Yield	Price	Yield Spread
Extendible	12.62	99.625	—
Straight "Short"	12.93	92.56	+0.31
Straight "Long"	12.79	91.75	+0.17

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
12.69	12.86	12.70	12.98

6. Relevant Notes (if any)

#6

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 12½'s 1 February '86-'91
   Amount Outstanding: 725
   Issue Date: 1 February '81

2. Straight Bond with Same Short Maturity: 10¾'s 1 October '85
   Amount Outstanding: 325
   Issue Date: 1 October '80

3. Straight Bond with Same Long Maturity: 11¼'s 15 December '89
   Amount Outstanding: 325
   Issue Date: 15 December '79

4. Yields at Issue Date of Extendible Bond (21 January '81):

	Yield	Price	Yield Spread
Extendible	12.79	98.94	—
Straight "Short"	12.96	92.38	+0.17
Straight "Long"	12.93	91.25	+0.14

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
13.06	13.02	12.83	12.96

6. Relevant Notes (if any)

#7

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 13¾'s 15 March '85-'90
   Amount Outstanding: 800
   Issue Date: 31 March '80

2. Straight Bond with Same Short Maturity: 11½'s 15 December '84
   Amount Outstanding: 700
   Issue Date: 15 December '79

3. Straight Bond with Same Long Maturity: 11½'s 15 December '89
   Amount Outstanding: 325
   Issue Date: 15 December '79 and subsequently

4. Yields at Issue Date of Extendible Bond (26 March '80):

	Yield	Price	Yield Spread
Extendible	13.66	100.31	—
Straight "Short"	13.86	92.00	+0.20
Straight "Long"	13.81	86.50	+0.15 (0.22)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
14.27	13.86	13.66	13.45

6. Relevant Notes (if any)

*If use "rule of thumb" that a 1% higher coupon corresponds to about 5 basis points, then yield spread rises to 22 basis points.

#8

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 13¼'s 1 February '85-'90
   Amount Outstanding: 600
   Issue Date: 31 March '81

2. Straight Bond with Same Short Maturity: 11¼'s 1 July '85
   Amount Outstanding: 450
   Issue Date: 1 July '80

3. Straight Bond with Same Long Maturity: 11¼'s 15 December '89
   Amount Outstanding: 450
   Issue Date: 15 December '79 and subsequently

4. Yields at Issue Date of Extendible Bond (25 March '81):

	Yield	Price	Yield Spread
Extendible	13.68	98.69	—
Straight "Short"	13.93	91.56	+0.25
Straight "Long"	13.69	87.75	+0.01 (+.11)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
16.22	15.51	15.09	14.96

6. Relevant Notes (if any)

*If assume that a 1% higher coupon corresponds to about 5 basis points, yield spread rises to 11 basis points.

#9

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: 15¼'s 1 June '86-'96
   Amount Outstanding: 900
   Issue Date: 1 June '81 & 31 July '81

2. Straight Bond with Same Short Maturity: 11¼'s 1 July '85
   Amount Outstanding: 450
   Issue Date: 1 June '80

3. Straight Bond with Same Long Maturity: 11¼'s 15 December '89
   Amount Outstanding: 450
   Issue Date: 15 December '79

4. Yields at Issue Date of Extendible Bond (27 May '81):

	Yield	Price	Yield Spread
Extendible	15.03	100.75	—
Straight "Short"	15.48	87.50	+0.45
Straight "Long"	15.18	81.50	+0.15 (+.35)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
16.22	15.51	15.09	14.96

6. Relevant Notes (if any)

*If assume that a 1% higher coupon requires a 5 basis point premium, then adjusted yield spread is 35 basis points.

#10

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. **Extendible Bond:** 14½'s 1 May '86-'91
   Amount Outstanding: 500
   Issue Date: 1 May '81

2. **Straight Bond with Same Short Maturity:** 11¼'s 1 July '85
   Amount Outstanding: 450
   Issue Date: 1 June '80

3. **Straight Bond with Same Long Maturity:** 11¼'s 15 December '89
   Amount Outstanding: 450
   Issue Date: 15 December '79

4. **Yields at Issue Date of Extendible Bond (29 April '81):**

	Yield	Price	Yield Spread	
Extendible	14.92	98.56	—	
Straight "Short"	15.44	87.44	+0.52	
Straight "Long"	14.98	82.25	+0.06	(+0.22)*

5. **Term Structure of Canada Yields at Issue Date of Extendible Bond:**

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
15.78	15.30	14.84	15.07

6. **Relevant Notes (if any)**

*If assume that a 1% higher coupon requires a 5 basis point premium, the yield spread rises to 22 basis points.

#11

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: $14\frac{3}{4}$'s 1 July '86-'93
   Amount Outstanding: 450
   Issue Date: 1 July '81

2. Straight Bond with Same Short Maturity: $11\frac{1}{4}$'s 1 July '85
   Amount Outstanding: 450
   Issue Date: 1 June '80

3. Straight Bond with Same Long Maturity: $11\frac{1}{4}$'s 15 December '89
   Amount Outstanding: 450
   Issue Date: 15 December '79

4. Yields at Issue Date of Extendible Bond (30 June '81):

	Yield	Price	Yield Spread
Extendible	15.09	98.81	—
Straight "Short"	15.72	87.12	+0.63
Straight "Long"	15.27	81.50	+0.18 (+0.36)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
16.19	15.52	15.24	15.03

6. Relevant Notes (if any)

*If assume that a 1% higher coupon requires a 5 basis point premium, the yield spread rises to 36 basis points.

#12

WORKSHEET FOR EXTENDIBLE GOVERNMENT OF CANADA BOND

1. Extendible Bond: $13\frac{3}{4}$'s 1 August '89
   Amount Outstanding:
   Issue Date:

2. Straight Bond with Same Short Maturity: $10\frac{1}{2}$'s 1 October '84
   Amount Outstanding: 300
   Issue Date: 1 October '79

3. Straight Bond with Same Long Maturity: $11\frac{1}{4}$'s 15 December '89
   Amount Outstanding: 325
   Issue Date: 15 December '79

4. Yields at Issue Date of Extendible Bond (25 February '81):

	Yield	Price	Yield Spread
Extendible	13.25	101.31	—
Straight "Short"	13.52	91.62	+0.27
Straight "Long"	13.52	88.50	+0.27 (+0.39)*

5. Term Structure of Canada Yields at Issue Date of Extendible Bond:

1-3 Years	3-5 Years	5-10 Years	Over 10 Years
13.66	13.48	13.32	13.38

6. Relevant Notes (if any)

*If assume that a 1% higher coupon requires 5 basis point premium, the yield spread rises to 39 basis points.

Appendix C

HOW THE TERM STRUCTURE OF INTEREST RATES AFFECTS THE VALUE OF THE PUT OPTION

To illustrate how the term structure of interest rates affects the value of the option which allows the issuer to extend the maturity of a bond, two simplifying assumptions are made. First, it is assumed that the pure expectations theory holds, so that the forward rate is an unbiased estimator of the expected rate; and second, that expectations hold with certainty. While such assumptions are extreme, they do provide a simple framework to develop the argument.

Consider a bond where in n years time, the bond issuer has the option to extend the maturity of the bond for m more years. If the option is not used, the bond matures at the end of year n. The extendable bond is initially issued with an interest rate of R_E. If it is extended, the interest rate is R'_E. At the end of year n, the bond issuer must decide whether to extend the maturity of the bond. If the bond's maturity is extended, let B_m denote the market value of the bond. The bond issuer will extend the maturity of the bond if B_m is less than the face value of the bond, F. The value of the option to extend is thus $F - B_m > 0$. At the end of year n, if an m-period bond is issued at par, then its coupon rate must equal the yield to maturity, R_m. Hence:

$$F - B_m = \sum_{j=1}^{m} \frac{(R_m - R'_E)F}{(1 + R_m)^j} \geq 0 \qquad \text{for} \qquad R_m - R'_E \geq 0$$

At the time the extendable bond is issued, given the assumption that the forward rate is an unbiased estimator of the expected rate of

interest, the value of the put option depends upon the spread between the then forward rate and the rate of interest on the bond when it is extended. That is, the value of the put option depends upon $(f - R'_E)$ where f is the appropriate forward rate. Clearly, the level of the deductible, the difference between R'_E and R_E, can be adjusted to make $f - R'_E$ and hence the value of the put option arbitrarily small. If the extendable rate R'_E equals the forward rate f at the date of issue of the extendable bond, then the value of the put option is zero.

In Figure 1 the term structure of interest rates is shown for different dates. For January 25, 1978, if 8.62% is taken for the rate of interest on a five year bond extendable for an additional five years, then by setting the interest rate R'_E equal to 8.89%, implying a deductible of .27%, the value of the put option will be zero. In January 26, 1983 a deductible of .98% is necessary for the value of the put to be zero. For a date like January 30, 1979 there is a downward sloping term structure of interest rates. Given the assumptions that have employed, this would imply that the option to extend the maturity would be worthless.

While the above argument demonstrates that the forward rate is an important factor affecting the value of the put option, there are other variables which will influence its value. For example, when the assumption that expectations are held with certainty is relaxed, then the uncertainty associated with unanticipated changes in interest rates will affect the value of the put option and thus the value of the extendable bond. Hence even for the term structure shown for the date January 30, 1979, the put option will still be of some value to the issuer.

Figure 1

Average Bond Yields*

	1 - 3 years	3 - 5 years	5 - 10 years	Over 10 years	5 year forward rate
1978 J25	7.68	8.36	8.62	9.06	8.89
1979 J30	10.05	9.88	9.81	9.82	9.74
1980 J30	12.79	12.49	12.11	12.13	11.73
1981 J28	13.06	13.02	12.83	12.96	12.64
1982 J27	15.93	15.73	15.95	15.94	16.17
1983 J26	9.89	10.19	11.17	12.28	12.15

The five year forward rate is calculated as $(1+R_{10})^{10} = (1+R_5)^5 (1+f)^5$, where R_5 is the 5 year interest rate (the 3-5 year rate is used), R_{10} is the 10 year interest rate (the 5-10 year rate is used); and f is the five year forward rate.

*Data source: Bank of Canada Review, various issues.

Appendix D

RETRACTABLE BONDS

A 5-year bond extendible for 5 years at the option of the issuer is equivalent to a straight 5-year bond less the value of a put option. A 10-year bond retractable to 5 years at the option of the holder is equivalent to a straight 10 year bond plus the value of a put option. Hence, assuming symmetry in the pricing of these options, 10-year retractable bonds can be used to price the put option.

As there are no Government of Canada retractable bonds currently outstanding, Provincial bonds have been used. However, it proved to be very difficult to find reliable data for the retractable bonds and the matched straight bonds.* The results are shown in Table D1. The first point to note is that the average length of the retraction feature is 10 years, twice the length envisaged by C.M.H.C. For the Saskatchewan bond, the amortized value of the put option is 139 basis points which, in comparison to the results in the rest of the study, is extremely large. The term structure of interest rates was upward sloping in September 1982 and thus the put option would be 'in the money'. The 12½ Canada's of '86 yielded 12.48 percent and the 13¼'s of '92 yielded 13.53 percent.

All of the results in Table 1 must be treated with caution as the market for these bonds is thin. In August 1982, the amortized value of the put option for Saskatchewan bond was 25 basis points. The term structure was again upward sloping. For the Quebec bond, the amortized value of the put option is 4 basis points. However, in May, 1982, the retractable bond was trading for $92.25 and the straight bond for $99.50. Given that the

*It should be recalled that a retractable bond with a put option can always be viewed as an extendible bond with a put option. The better quality of data available for Government of Canada extendible bonds, than for Provincial retractables, implies that the results for the extendible bonds are more reliable.

retractable bond has a higher coupon and an option feature, it should always sell for more than the straight bond.

Table D1

Retractable Bonds

RETRACTABLE BOND			STRAIGHT BOND			Observation Date	Yield Difference
Name	Coupon	Maturity	Name	Coupon	Maturity		
Saskatchewan	$13\frac{3}{8}$	'96-'86	Saskatchewan	11.75	.00	September '82	139
Nova Scotia Municipal Finance	15	'96-'86	Nova Scotia Power	13.5	.02	October '82	26
Quebec	13.25	'93-'84	Quebec	12.75	'92	November '82	4

DATE: _____

Appendix E

ESTIMATING THE PREMIUM REQUIRED TODAY ON A HYPOTHETICAL BOND
EXTENDIBLE AT THE OPTION OF THE GOVERNMENT OF CANADA

BOND #1 5-Year, Extendible at Issuer's Option for 5 Years

	Yield on 5-Year Bond Extendible for 5-Years at		
Yield on Identical 5-Year Bond	Initial Rate	Initial Rate + 1%	Initial Rate + 3%
_____	_____	_____	_____

BOND #2 1-Year, Extendible at Issuer's Option for 1 Year

	Yield on 1-Year Bond Extendible for 1-Year at		
Yield on Identical 1-Year Bond	Initial Rate	Initial Rate + 1%	Initial Rate + 3%
_____	_____	_____	_____

1. Over a period of years, what is the likely range in the size of the premiums that you might expect to observe?

	5-Year Bond Extendible at	1-Year Bond Extendible at
Initial Rate	_____	_____
Initial Rate + 3%	_____	_____

2. How important is issue size?

3. Who are the most likely buyers of such an issue?

4. Is there a particular maturity for the extendible bonds that potential buyers might prefer?

5. Are there any general comments you would like to make regarding the likely market reception to such bonds?

Please return by April 28 to
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POOR

QUALITY

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